



A Review of Lossless Image Compression Based On Transform Function

Avisha Khanna and Sanjeev Sharma

Department of Electronics and Communication Engineering,

SOIT, RGPV, Bhopal, (MP)

(Received 17 September, 2011, Accepted 14 October, 2011)

ABSTRACT : The growth of data rate is increases in decades. The maximum part of data is image, image required huge amount of memory for storage purpose and take more time for process of storage. Now lossless image compression is a necessary for saving memory and time. In the process of lossless image compression transform function play a vital role. In this paper we discuss various transform function used for lossless image compression such as DCT, DPCM, WPT and IWPT.

Keywords: Image Compression, DCT, DPCM, WPT, IWPT.

I. INTRODUCTION

Lossless image compression plays a crucial rule in the filed of data compression. It develops very slowly and it is difficult to improve the efficiency of compression, since complete information and high fidelity are demanded. The lossless compression ratio of the conventional methods is around 2 : 1 and 3 : 1, which doesn't satisfy practical compression needs. So, it is necessary to develop more efficient lossless image compression methods [5]. The standard methods of image compression come in several varieties. The current most popular method relies on eliminating high frequency components of the signal by storing only the low frequency components. This method is used on JPEG, MPEG, H.261, and H.263 compression algorithms. in this paper we discuss image compression based on transform function. A transfer function is a mathematical representation, in terms of spatial or temporal frequency, of the relation between the input and output of a linear time-invariant system. With optical imaging devices, for example, it is the Fourier transform of the point spread function (hence a function of spatial frequency) i.e. the intensity distribution caused by a point object in the field of view. Transfer functions are commonly used in the analysis of systems such as single-input single-output filters, typically within the fields of signal processing, communication theory, and control theory. The discrete cosine transform (DCT) is a technique for converting a signal into elementary frequency components. It is widely used in image compression. DPCM or differential pulse-code modulation is a signal encoder that uses the baseline of PCM but adds some functionality based on the prediction of the samples of the signal [6]. The input can be an analog signal or a digital signal. The discrete wavelet transform is a subset of the far more versatile wavelet packet transform, which generalizes the time-frequency analysis of the wavelet

transform. Integer wavelet packet transform function is a whole number part of low pass filter and get integer number such number is called packet. This paper is organized as follows. In Section II, DCT (Discrete cosine Transform). III In this Section DPCM (differential pulse-code modulation). IV in this section discusses WPT-IWPT (wavelet packet transforms). In section discuss conclusion of transform function in image compression.

II. DCT (DISCREET COSINE TRANSFORM)

The discrete cosine transform of a list of n real numbers $s(x)$, $x = 0, n - 1$, is the list of length n given by [2]:

$$S(u) = \sqrt{\frac{2}{n}} C(u) \sum_{x=0}^{n-1} s(x) \cos(2x+1)u \prod / 2n$$

$$u = 0, \dots, n,$$

where $c(u) = 2 - 1/2$ for $u = 0 = 1$ otherwise

Each element of the transformed list $S(u)$ is the inner (dot) product of the input list $s(x)$ and a *basis vector*. The constant factors are chosen so that the basis vectors are orthogonal and normalized. The DCT can be written as the product of a vector (the input list) and the $n \times n$ orthogonal matrix whose rows are the basis vectors. This matrix, for $n = 8$, can be computed as follows :

DCTMatrix =

Table[If[k==0,

Sqrt[1/8],

Sqrt[2/8] Cos[Pi (2j + 1) k/16]],

{k, 0, 7}, {j, 0, 7}] // N;

We can check that the matrix is orthogonal :

DCTMatrix . Transpose [DCTMatrix] // Chop// Matrix Form [2]

| | | | | | | |
|----|----|----|----|----|----|----|
| 1. | 0 | 0 | 0 | 0 | 0 | 0 |
| | 0 | | | | | |
| 0 | 1. | 0 | 0 | 0 | 0 | 0 |
| | 0 | | | | | |
| 0 | 0 | 1. | 0 | 0 | 0 | 0 |
| | 0 | | | | | |
| 0 | 0 | 0 | 1. | 0 | 0 | 0 |
| | 0 | | | | | |
| 0 | 0 | 0 | 0 | 1. | 0 | 0 |
| | 0 | | | | | |
| 0 | 0 | 0 | 0 | 0 | 1. | 0 |
| | 0 | | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 1. |
| | 0 | | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | 1. | | | | | |

Each basis vector corresponds to a sinusoid of a certain frequency. The eight basis vectors for the discrete cosine transform of length eight. The list $s(x)$ can be recovered from its transform $S(u)$ by applying the inverse cosine transform (IDCT):

$$S(u) = \sqrt{\frac{2}{n}} C(u) \sum_{x=0}^{n-1} s(x) \cos(2x+1)u\pi / 2n$$

$$u = 0, \dots, n,$$

where $c(u) = 2 - 1/2$ for $u = 0 = 1$ otherwise.

This equation expresses s as a linear combination of the basis vectors. The coefficients are the elements of the transform S , which may be regarded as reflecting the amount of each frequency present in the input s . This is a linear discrete cosine transform function used in frequency spatial domain in image compression.

III. DPCM (DIFFERENTIAL PULSE CODE MODULATION)

Differential Pulse Code Modulation (DPCM) is transformation for increasing the compressibility of an image. It consists of scanning the image and predicting the next pixel's value. There are several modes to predict the next pixel's value. The basic idea behind this scheme is to predict the value of a pixel based on certain neighboring pixel values, using certain prediction coefficients [3]. The difference between the predicted value and actual value of the pixels is differential image, which is much less correlated than the original image. The differential image is then quantized and encoded. The schematic for loss DPCM coder is shown in Fig. 1 [14], along with a third order predictor. Note that the decoder has access only to the reconstructed values of pixel while forming predictions of

pixels. Since the quantization of the differential image introduces error, the reconstructed values generally differ from the original values. To ensure identical predictions at both the encoder and decoder, the encoder also uses the reconstructed pixel values in its predictions. The design of a DPCM coder involves the optimization of the predictor and the quantizer. The inclusion of the quantizer in the prediction loop results in a complex dependency between the predictor error and the quantization error [14].

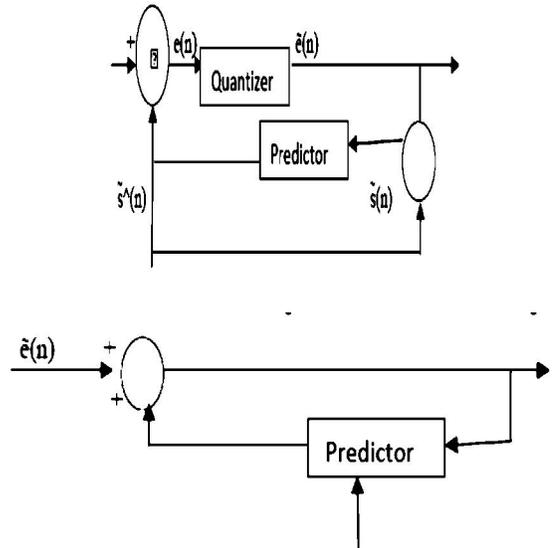


Fig. 1. The encoder and decoder of DPCM.

Here we give a complete step for algorithm for image compression based on DPCM [14].

- I : Original image.
- \hat{i} : Reconstructed image.
- $N \times M$: Image size (N rows, M columns of pixels).
- a, b, c : Prediction coefficients.
- E^* : Quantized residual image.
- $CR[1 : M]$: Current row of reconstructed values.
- $UR[1 : M]$: Upper row of reconstructed values.
- $LC[1 : N]$: Leftmost column of reconstructed values.
- Q : Quantizer function $-Q(x)$ gives the quantized value of x .
- Q^{-1} : Dequantizer.
- B : Entropy coded bit stream.
- k : Number of quantization levels.
- DV : Vector of decision levels.
- RV : Vector of reconstruction levels.

Algorithm Lossy DPCM_Encode (in: I, a, b, c, Q , out:).

1. begin
2. $E^*[1, 1] \leftarrow Q(I[1, 1])$
3. $LC[1] \leftarrow CR[1] \quad Q^{-1}(E^*[1, 1])$
 {Work on top row}
4. for $j = 2$ to M do
5. $E^*[1, j] \leftarrow Q(I[1, j] - CR[j - 1])$

```

6.  $CR[j] \leftarrow Q^{-1}(E^*[1, j]) + CR[j - 1]$ 
7. endfor
   {Work on leftmost column}
8. for  $i = 2$  to  $N$  do
9.  $E^*[i, 1] \leftarrow Q(I[i, 1] - LC[i - 1])$ 
10.  $LC[i - 1] \leftarrow Q^{-1}(E^*[i, 1] + LC[i - 1])$ 
11. endfor
12. for  $i = 2$  to  $N$  do
13.  $UR[1 : m] \leftarrow CR[1 : M]$ 
14.  $CR[1] \leftarrow LC[i]$ 
15. for  $j = 2$  to  $M$  do
16.  $p \leftarrow a \cdot CR[j - 1] + b \cdot UR[j - 1] + c \cdot UR[j]$ .
{Prediction}
17.  $E^*[i, j] \leftarrow Q(I[i, j] - p)$ 
18.  $CR[j] \leftarrow Q^{-1}(E^*[i, j]) + p$ 
19. endfor
20. endfor
21.  $B \leftarrow \text{ENTROPY\_CODE}(E^*)$ 
22. end
Algorithm Lossy DPCM_Decode (in: B, a, b, c, Q, out:  $\hat{I}$ )
1. begin
2.  $E^* \leftarrow \text{ENTROPY\_DECODE}(B)$ 
3.  $\hat{I}[1, 1] \leftarrow Q^{-1}(E^*[1, 1])$ 
   {Work on top row}
4. for  $j = 2$  to  $M$  do
5.  $\hat{I}[1, j] \leftarrow Q^{-1}(E^*[1, j] + \hat{I}[1, j - 1])$ 
6. endfor
   {Work on leftmost column}
7. for  $i = 2$  to  $N$  do
8.  $\hat{I}[i, 1] \leftarrow Q^{-1}(E^*[i, 1] + \hat{I}[i - 1, 1])$ 
9. endfor
10. for  $i = 2$  to  $N$  do
11. for  $j = 2$  to  $M$  do
12.  $\hat{I}[i, j] \leftarrow Q^{-1}(E^*[i, j] + a\hat{I}[i, j - 1] + b\hat{I}[i - 1, j - 1] + c\hat{I}[i - 1, j])$ 
13. endfor
14. endfor
15. end

```

IV. WPT-IWPT (WAVELET PACKET TRANSFORM AND INTEGER WAVELET PACKET TRANSFORM)

Wavelet packets represent a generalization of multiresolution decomposition. The block scheme of the single level wavelet transform is shown in Fig. 2.

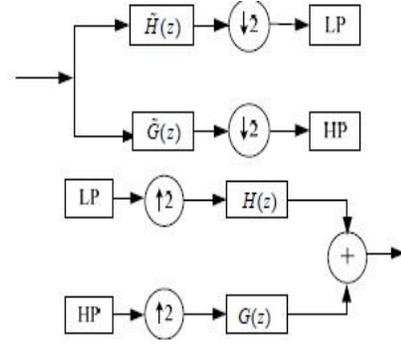


Fig. 2. One level wavelet transforms.

The low-pass analysis filters and the high-pass ones are followed by down sampling of a factor two. At the reconstruction side, the low-pass and band-pass branches are up sampled and filtered with the synthesis filters $H(z)$ and $G(z)$ in order to obtain the original signal. A wavelet transform on J levels is obtained by iterating the filter bank $J - 1$ times on the low-pass branch. The wavelet transform coefficients consist of the J high-pass and the terminal low-pass node sequences output by the filter bank tree. Given a perfect reconstruction filter bank, the iterated scheme represents an either orthonormal or biorthogonal (non redundant) representation of the original signal. Differently from the wavelet transform, the J -level WPT are achieved by iterating the one level filter bank on both the low-pass and the high-pass branch, and then applying a pruning algorithm to select a suitable representation. An algorithm has been proposed in [12], which selects the best representation of a sequence across the entire tree based on some proper cost function, which must measure the compactness of the representation.

The Lifting Scheme

In this section we give a brief description of the LS, which serves to introduce the LS-based IWPT. The mathematical formulation follows closely the one in [11]. Within the LS framework it is useful to represent digital filters by means of their polyphase representation, and the polyphase representation of a filter $H(z)$ is described as following :

$$H(z) = H_e(z^2) + z^{-1}H_o(z^2) \quad \dots (1)$$

where $H_e(z)$ and $H_o(z)$ are respectively obtained from the even and odd coefficients of $h(n) = Z^{-1}[H(z)]$, where Z denotes the zeta transform. The wavelet filter bank can be expressed in polyphase formulation defining a polyphase matrix as

$$P(z) = \begin{bmatrix} H_e(z) & G_e(z) \\ H_o(z) & G_o(z) \end{bmatrix} \quad \dots (2)$$

For the synthesis filters, and analogously $P(z)$ for the

analysis ones. Two filters $[H(z), G(z)]$ are said to be complementary. Primal Lifting in [11] states that, if $H(z)$ and $G(z)$ are complementary, then any finite filter of the form $G_{new}(z) = G(z) + H(z)s(z^2)$, with $s(z)$ a Laurent polynomial (i.e. the z -transform of a FIR filter), is complementary to $H(z)$. Moreover, Dual Lifting states that any finite filter of the form $H_{new}(z) = H(z) + G(z)t(z^2)$, with $t(z)$ a Laurent polynomial, is complementary to $G(z)$. Therefore, if the primal and dual lifting steps are applied to an initial analysis filter bank, new orthonormal or biorthogonal representations can be derived. It can be shown that any known wavelet transform can be obtained by means of the LS, starting from a separation of the even and odd coefficients (the lazy wavelet), and ending with a normalization. The two lifting steps are depicted in Fig. 3 and Fig. 4 respectively.

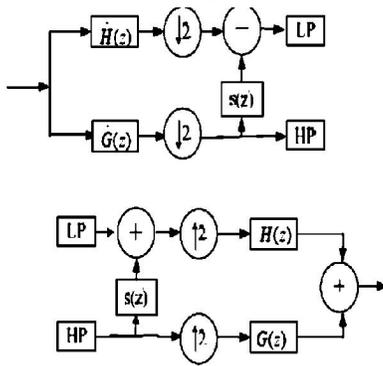


Fig. 3. Primal lifting.

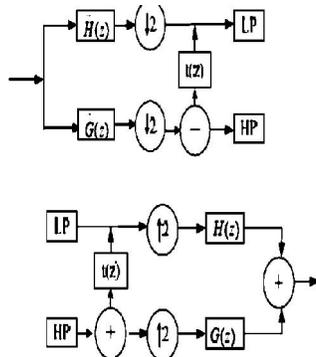


Fig. 4. Dual lifting.

A technique for the factorization of the polyphase matrix using the Euclidean algorithm for the greatest common divider between two Laurent polynomials is described in [11]. From this factorization a scheme follows, which achieves a wavelet transform as sequence of alternated primal and dual lifting steps. The inverse wavelet transform is achieved with the proper sequence of inverse steps. The complexity can be halved with respect to that of the filter bank scheme. Omitting details, it has been proven that rounding off the output of each filter right before adding or subtracting yields a couple of perfect reconstruction forward and inverse IWT. It is straightforward to understand that the same procedure which leads to IWT from wavelet transform can be applied to the wavelet packets transform, yielding the IWPT. The implementation follows the same scheme used for the IWT. The IWPT tree can be built iterating the single wavelet decomposition step on both

the low-pass and high-pass branches, with rounding off in order to achieve the integer transforms. IWPT yields a representation which can be lossless, as it maps an integer valued sequence onto integer valued coefficients in the Transformed domain; moreover, it allows for the selection of an adaptive representation, which can match the variable characteristics of image better than the IWT.

V. CONCLUSION AND FUTURE WORK

Image compression a challenging filed of image processing. In this paper we discuss various transform method for lossless image compression such as DCT, DPCM, WPT-IWT. These entire method compression ratios are very high. In this paper we discuss a decomposition process of wavelet packet transform and also discuss a linear discreate cosine transform function for simple image compression. In future we combined a two different transform function and perform another method for an image compression.

REFERENCES

- [1] Hua Li. And Yiming Zhu "Lossless Image Compression Based On DPCM-IWPT" in (2008) ISECS International Colloquium on Computing, Communication, Control, and Management.
- [2] Andrew B. Watson "Image Compression Using the Discrete Cosine Transform" in *Mathematica Journal*, **4**(1), (1994), p. 81-88.
- [3] Shilpa s. Dhulap" Image Compression Based On Iwt, Iwpt & Dpcm-Iwpt" in *International Journal of Engineering Science and Technology* Vol. **2**(12), (2010), 7413-7422.
- [4] R.C. Calderbank, I. Daubechies, W. Sweldens, and B. Yeo, "Lossless Image Compression using Integer to Integer Wavelet Transforms", *International Conference on Image Processing*, Vol. **1**, pp. 596-599.
- [5] S. Dewine and Comelis, "Lossless integer wavelet transform". *IEEE Signal Pmcessing Mer*, (1997), Vol. **4**, pp. 158-150.
- [6] Adrian Munteanu, etal, "Wavelet-based Lossless Compression of Coronary Angiographic Images". *IEEE Transactions on Medical Imaging*, (1999), Vol. **18** no. 3, pp. 272-281.
- [7] K. Ramchandran and M. Vetterli, "Best wavelet packet bases in a rate-distortion sense", *IEEE Trans. Image Processing*, (1993), Vol. **2**, no. 2, pp. 160-175.
- [8] K. Ramchandran, M. Vetterli, and C. Herley, "Wavelets, subband coding, and best bases", *Proceedings of The IEEE*, (1996), Vol. **84**, no. 4, pp. 541-560.
- [9] Vetterli M., Kova^cevi'c J., "Wavelets and subband coding", Prentice Hall PTR, Englewood Cliffs, New Jersey, (1995).
- [10] Sweldens W., "The lifting scheme: A construction of second generation wavelets", *Siam J. Math. Anal*, (1997), Vol. **29**, no. 2, pp. 511-546.
- [11] I. Daubechies, W. Sweldens, "Factoring Wavelet Transforms into Lifting Steps", *J. Fourier Anal. Appl.*, (1998), Vol. **4**, no. 3, pp. 247-269.
- [12] Coifman R. R., Wickerhauser M. V., "Entropy-based algorithms for best basis selection", *IEEE Trans. On Information theory*, (1992), Vol. **38** no. 2, pp. 713-718.
- [13] R.C. Calderbank, I. Daubechies, W. Sweldens, and B. Yeo, "Wavelet Transforms that Map Integers to Integers", *Applied and Computational Harmonic Analysis*, (1998), Vol. **5**, no. 3, pp. 332-369, 160.
- [14] S.R. subramanya and Abdou Youssef "Performance evaluation of lossy DPCM coding of images using dirrerent predictors and quantizers" 1967, vol. **30**, pp. 483-485.